



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc., DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2013

MT 1501 – GRAPHS, DIFF. EQU., MATRICES & FOURIER SERIES

Date : 14/11/2013

Dept. No.

Max. : 100 Marks

Time : 1:00 - 4:00

PART – A

Answer **ALL** questions:

(10 X 2 = 20)

1. Let $R \rightarrow R$ be defined by $f(x) = x^2$. Find the range of the function.
2. Find the slope of the line $x = -2y - 7$.
3. Write the normal equation of $y = a + bx$.
4. Reduce $y = ae^{bx}$ to normal form.
5. Define difference equation with example.
6. Solve: $y_{x+2} - 8y_{x+1} + 15y_x = 0$.
7. Show that the matrix $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is orthogonal.
8. Find the eigen values of the matrix $\begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.
9. Write down the Fourier series for the function $f(x)$ defined in the interval $0 < x < 2\pi$.
10. Define odd and even functions with examples.

PART – B

Answer any **FIVE** questions:

(5 X 8 = 40)

11. A steel plant produces x tons of steel per week at a total cost of Rs. $(\frac{1}{3}x^3 - 5x^2 + 99x + 35)$. Find the output level at which the marginal cost attains its minimum.
12. Let the cost function of a firm be given by $C(x) = 300x - 10x^2 + \frac{1}{3}x^3$ for x units. Find x at which the average cost is minimum.
13. Using method of least squares, fit a straight line for the following data.
Estimate y when $x = 25$.

x	0	5	10	15	20
y	7	11	16	20	26

14. Solve: $y_{x+2} - 6y_{x+1} + 8y_x = 4^x$.

15. Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$

16. Find the eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

17. Obtain the Fourier expansion of $f(x) = \frac{1}{2}(\pi - x)$, $(0 < x < 2\pi)$.

18. Obtain the half range cosine series for the function $f(x) = x$, $(0 < x < \pi)$.

PART-C

Answer any **TWO** questions:

(2 X 20 = 40)

19. a. Fit a curve of the form $y = a + bx + cx^2$ for the following table.

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

b. The cost function for producing x units of a product is $C(x) = x^3 - 12x^2 + 48x + 11$ (in rupees) and the revenue function is $R = 83x - 4x^2 - 21$. Find the output for which the profit is maximum. Find also the maximum profit.

20. a. Solve the difference equation: $(x + 2) - 4u(x) = 9x^2$.

b. Solve the equation $y_{n+2} - 16y_n = \cos \frac{1}{2}n$.

21. Determine the Fourier expansion for $f(x) = \begin{cases} -\pi & \text{in } (0, \pi) \\ x - \pi & \text{in } (\pi, 2\pi) \end{cases}$

and show that $\sum_{r=1}^{\infty} \frac{1}{(2r+1)^2} = \frac{\pi^2}{8}$.

22. Diagonalise the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.

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